Fișa numărul 102.

Exercițiul 1. Scoateți factorii de sub radical

a) $\sqrt{50}$; $\sqrt{28}$; $\sqrt{27}$; $\sqrt{80}$; $\sqrt{75}$.

b) $\sqrt{24}$; $\sqrt{96}$; $\sqrt{150}$; $\sqrt{250}$; $\sqrt{60}$.

c) $\sqrt{216}$; $\sqrt{294}$; $\sqrt{360}$; $\sqrt{810}$; $\sqrt{540}$.

d) $\sqrt{960}$; $\sqrt{726}$; $\sqrt{605}$; $\sqrt{500}$; $\sqrt{250}$.

Rezolvare

$\sqrt{a^2 \cdot b} = a\sqrt{b}$, $a, b \geq 0$.

a) $\sqrt{50} = \sqrt{5^2 \cdot 2} = 5\sqrt{2}$

50	2
25	5
5	5
1	

$\sqrt{28} = \sqrt{2^2 \cdot 7} = 2\sqrt{7}$.

28	2
14	2
7	7
1	

$\sqrt{27} = \sqrt{3^3} = \sqrt{3^2 \cdot 3} = 3\sqrt{3}$.

$\sqrt{80} = \sqrt{2^4 \cdot 5} = 2^2\sqrt{5}$

$= 4\sqrt{5}$.

80	2
40	2
20	2
10	2
5	5
1	

$\sqrt{75} = \sqrt{5^2 \cdot 3} = 5\sqrt{3}$.

75	5
15	5
3	3
1	

b) $\sqrt{24} = \sqrt{4 \cdot 6} = 2\sqrt{6}$.

$\sqrt{96} = \sqrt{2^5 \cdot 3} = \sqrt{2^4 \cdot 2 \cdot 3}$

$= 2^2\sqrt{6}$

$= 4\sqrt{6}$.

96	2
48	2
24	2
12	2
6	2
3	3
1	

$\sqrt{150} = \sqrt{15 \cdot 10} = \sqrt{3 \cdot 5 \cdot 5 \cdot 2}$

$= 5\sqrt{6}.$

$\sqrt{250} = \sqrt{25 \cdot 10} = 5\sqrt{10}$

$\sqrt{60} = \sqrt{2 \cdot 3 \cdot 2 \cdot 5} = \sqrt{2^2 \cdot 3 \cdot 5} = 2\sqrt{15}$

c) $\sqrt{216} = \sqrt{6^3} = 6\sqrt{6}$

$\sqrt{294} = \sqrt{7^2 \cdot 2 \cdot 3} = 7\sqrt{6}$

294	2
147	3
49	7^2

$\sqrt{360} = \sqrt{6^2 \cdot 10} = 6\sqrt{10}.$

$\sqrt{810} = \sqrt{9^2 \cdot 10} = 9\sqrt{10}$

$\sqrt{540} = \sqrt{2^2 \cdot 3^3 \cdot 5} = 2 \cdot 3\sqrt{15}$

$= 6\sqrt{15}$

540	2
270	3
90	3
30	3
10	2
5	5
1	

d) $\sqrt{960} = \sqrt{2^6 \cdot 3 \cdot 5} = 2^3\sqrt{15}$

$= 8\sqrt{15}$

960	2
480	2
240	2
120	2
60	2
30	2
15	3
5	5
1	

$\sqrt{726} = \sqrt{11^2 \cdot 2 \cdot 3} = 11\sqrt{6}.$

726	2
363	3
121	11
11	11
1	

$\sqrt{605} = \sqrt{11^2 \cdot 5} = 11\sqrt{5}.$

605	5
121	11
11	11
1	

$\sqrt{500} = \sqrt{5 \cdot 100} = \sqrt{5 \cdot 10^2} = 10\sqrt{5}$

$\sqrt{250} = \sqrt{5^2 \cdot 10} = 5\sqrt{10}$

Exercitiul 2. Scoateți factorii de sub radical:

a) $\sqrt{2^2 \cdot 3^2 + 3\sqrt{576}}$ b) $\sqrt{2^3 + 4\sqrt{324}}$ c) $\sqrt{2^4 + 2\sqrt{369}}$

d) $\sqrt{33 + 6\sqrt{361}}$ e) $\sqrt{2^2 \cdot 3 + 8\sqrt{441}}$ f) $\sqrt{3^2 + 3^2\sqrt{121}}$

Rezolvare

a) $\sqrt{2^2 \cdot 3^2 + 3\sqrt{576}} = \sqrt{2^2 \cdot 3^2 + 3 \cdot 24} = \sqrt{2^2 \cdot 3^2 + 3 \cdot 2^3 \cdot 3}$

$= \sqrt{2^2 \cdot 3^2 \cdot (1 + 2)}$

$= 2 \cdot 3\sqrt{3}$

$= 6\sqrt{3}$

b) $\sqrt{2^3 + 4\sqrt{324}} = \sqrt{2^3 + 4 \cdot 18} = \sqrt{2^3 + 4 \cdot 3^2 \cdot 2}$

$= \sqrt{2^3(1 + 3^2)}$

$= \sqrt{2^3 \cdot 10} = \sqrt{2^2 \cdot 2 \cdot 2 \cdot 5} \Rightarrow$

$= 2 \cdot 2\sqrt{5} = 4\sqrt{5}$

c) $\sqrt{2^4 + 2\sqrt{169}} = \sqrt{2^4 + 2 \cdot 13}$

$= \sqrt{16 + 26} = \sqrt{42} = \sqrt{2 \cdot 21} = \sqrt{2 \cdot 3 \cdot 7}$

d) $\sqrt{33 + 6\sqrt{361}} = \sqrt{33 + 6 \cdot 19} = \sqrt{3 \cdot 11 + 6 \cdot 19}$

$= \sqrt{3(11 + 2 \cdot 19)}$

$= \sqrt{3 \cdot (11 + 38)}$

$= \sqrt{3 \cdot 49} = 7\sqrt{3}$

e) $\sqrt{2^2 \cdot 3 + 8\sqrt{441}} = \sqrt{2^2 \cdot 3 + 8 \cdot 21} = \sqrt{2^2 \cdot 3(1 + 2 \cdot 7)}$

$= \sqrt{2^2 \cdot 3 \cdot 15} = \sqrt{2^2 \cdot 3 \cdot 3 \cdot 5}$

$= 2 \cdot 3\sqrt{5} = 6\sqrt{5}.$

f) $\sqrt{3^2+3^2\sqrt{121}} = \sqrt{3^2+3^2\cdot 11} = \sqrt{3^2(1+11)}$

$= 3\sqrt{12}$

$= 3\sqrt{2^2\cdot 3} = 3\cdot 2\sqrt{3} = 6\sqrt{3}$

Exercitiul 3. Să se calculeze:

a) $\sqrt{2^4\cdot 3^2}$; $\sqrt{2^6\cdot 3^2\cdot 5^2}$; $\sqrt{3^2\cdot 7^2}$

b) $\sqrt{5^2\cdot 3^6}$; $\sqrt{3^2\cdot 5^2\cdot 2^2}$; $\sqrt{3^4\cdot 7^2}$

c) $\sqrt{2^{10}\cdot 5^2}$; $\sqrt{3^2\cdot 2^8}$; $\sqrt{11^2\cdot 5^2}$

Rezolvare:

a) $\sqrt{2^4\cdot 3^2} = 2^2\cdot 3 = 4\cdot 3 = 12$

$\sqrt{2^6\cdot 3^2\cdot 5^2} = 2^3\cdot 3\cdot 5 = 8\cdot 3\cdot 5 = 24\cdot 5 = 120$

$\sqrt{3^2\cdot 7^2} = 3\cdot 7 = 14$

b) $\sqrt{5^2\cdot 3^6} = 5\cdot 3^3 = 5\cdot 27 = 135$

$\sqrt{3^2\cdot 5^2\cdot 2^2} = 3\cdot 5\cdot 2 = 30.$

$\sqrt{3^4\cdot 7^2} = 3^2\cdot 7 = 9\cdot 7 = 63.$

c) $\sqrt{2^{10}\cdot 5^2} = 2^5\cdot 5 = 32\cdot 5 = 160.$

$\sqrt{3^2\cdot 2^8} = 3\cdot 2^4 = 3\cdot 16 = 48.$

$\sqrt{11^2\cdot 5^2} = 11\cdot 5 = 55.$

Exercitiul 4. Calculați:

$\sqrt{14^2}$; $\sqrt{23^2}$; $\sqrt{(-26)^2}$; $\sqrt{(-35)^2}$; $\sqrt{3^6}$; $\sqrt{2^{10}}$; $\sqrt{(-7)^4}$; $\sqrt{625}$;

$\sqrt{1296}$; $\sqrt{2304}$.

Rezolvare:

$\sqrt{14^2} = 14$; $\sqrt{23^2} = 23$; $\sqrt{(-26)^2} = |-26| = +26$

$\sqrt{(-35)^2} = |-35| = +35$; $\sqrt{3^6} = 3^3 = 27$; $\sqrt{2^{10}} = 2^5 = 32$

$\sqrt{(-7)^4} = (-7)^2 = +49$; $\sqrt{625} = 25$; $\sqrt{1296} = 36$.

$\sqrt{2304} = \sqrt{48^2} = 48$.

Exercițiul 5. Să se efectueze calculele:

a) $\sqrt{16^2 \cdot 15^2}$; $\sqrt{18^2 \cdot 7^2 \cdot 2^4}$; $\sqrt{3^4 \cdot 5^2 \cdot 2^6}$; $\sqrt{2^8 \cdot 3^4 \cdot 7^2}$

b) $\sqrt{\frac{15^2}{23^2}}$; $\sqrt{\frac{2^4 \cdot 3^2}{5^4}}$; $\sqrt{\frac{7^2 \cdot 2^2}{3^4}}$; $\sqrt{\frac{25^2}{13^2 \cdot 2^2}}$; $\sqrt{\frac{2^2 \cdot 3^4 \cdot 5^2}{7^2 \cdot 11^2}}$

c) $\sqrt{12^2+5^2}$; $\sqrt{36^2+48^2}$; $\sqrt{15^2-12^2}$; $\sqrt{15^2+20^2}$; $\sqrt{30^2-24^2}$

d) $\sqrt{\frac{35^2-28^2}{12^2+9^2}}$; $\sqrt{\frac{24^2+18^2}{25^2-15^2}}$; $\sqrt{\frac{20^2-12^2}{21^2+28^2}}$

Rezolvare:

Observație $\sqrt{a^2 \pm b^2} \neq \sqrt{a^2} \pm \sqrt{b^2}$

$\sqrt{a^2} = a$, $a \geq 0$.

$\sqrt{a^2} = |a|$, pentru orice $a \in \mathbb{Q}$.

a) $\sqrt{16^2 \cdot 15^2} = 16 \cdot 15 = 240$

$\sqrt{18^2 \cdot 7^2 \cdot 2^4} = 18 \cdot 7 \cdot 4 = 504$

$\sqrt{3^4 \cdot 5^2 \cdot 2^6} = 9 \cdot 5 \cdot 8 = 360$.

$\sqrt{2^8 \cdot 3^4 \cdot 7^2} = 2^4 \cdot 3^2 \cdot 7 = 16 \cdot 9 \cdot 7 = 1008$.

b) $\sqrt{\frac{15^2}{23^2}} = \frac{15}{23}$; $\sqrt{\frac{2^4 \cdot 3^2}{5^4}} = \frac{4 \cdot 3}{25} = \frac{12}{25}$

$\sqrt{\frac{7^2 \cdot 2^2}{3^4}} = \frac{7 \cdot 2}{9} = \frac{14}{9}$; $\sqrt{\frac{25^2}{13^2 \cdot 2^2}} = \frac{25}{13 \cdot 2} = \frac{25}{26}$

$\sqrt{\frac{2^2 \cdot 3^4 \cdot 5^2}{7^2 \cdot 11^2}} = \frac{2 \cdot 9 \cdot 5}{7 \cdot 11} = \frac{90}{77}$.

c) $\sqrt{12^2+5^2} = \sqrt{144+25} = \sqrt{169} = 13.$

$\sqrt{36^2+48^2} = \sqrt{1296+2304} = \sqrt{3600} = \sqrt{36\cdot 100} = 6\cdot 10 = 60$

$\sqrt{15^2-12^2} = \sqrt{225-144} = \sqrt{81} = 9$

$\sqrt{15^2+20^2} = \sqrt{225+400} = \sqrt{625} = 25.$

$\sqrt{30^2-24^2} = \sqrt{900-576} = \sqrt{324} = 18.$

d) $\sqrt{\frac{35^2-28^2}{12^2+9^2}} = \sqrt{\frac{1225-784}{144+81}} = \sqrt{\frac{441}{225}} = \frac{21}{15} \overset{(3}{=} \frac{7}{5}$

$\sqrt{\frac{24^2+18^2}{25^2-15^2}} = \sqrt{\frac{576+324}{625-225}} = \sqrt{\frac{900}{400}} = \frac{30}{20} \overset{(10}{=} \frac{3}{2}.$

$\sqrt{\frac{20^2-12^2}{21^2+28^2}} = \sqrt{\frac{400-144}{441+784}} = \sqrt{\frac{256}{1225}} = \frac{16}{35}$

Exercitiul 6. Să se efectueze calculele:

a) $\sqrt{3^4\cdot(5^2-2^4)}$; $\sqrt{2^6\cdot(5^2-3^2)}$

b) $\sqrt{7^2\cdot 2^6+7^2\cdot 2^2\cdot 3^2}$; $\sqrt{5^2\cdot 2^4+5^2\cdot 3^2}$

c) $\sqrt{3^2\cdot 5^2+2^6}$; $\sqrt{2^4+2^3\cdot 3+3^2}$

d) $\sqrt{3^4\cdot 13^2-3^4\cdot 5^2}$; $\sqrt{5^2\cdot 17^2-2^6\cdot 5^2}$

Rezolvare

a) $\sqrt{3^4\cdot(5^2-2^4)} = \sqrt{3^4\cdot(25-16)} = \sqrt{3^4\cdot 9} = 3^2\cdot 3 = 27.$

$\sqrt{2^6\cdot(5^2-3^2)} = \sqrt{2^6\cdot(25-9)} = \sqrt{2^6\cdot 16} = 2^3\cdot 4 = 8\cdot 4 = 32$

b) $\sqrt{7^2 \cdot 2^6 + 7^2 \cdot 2^2 \cdot 3^2} = \sqrt{7^2 \cdot 2^2 (2^4 + 3^2)} = \sqrt{7^2 \cdot 2^2 \cdot (16+9)}$

$= \sqrt{7^2 \cdot 2^2 \cdot 25} = 7 \cdot 2 \cdot 5$

$= 14 \cdot 5 = 70.$

$\sqrt{5^2 \cdot 2^4 + 5^2 \cdot 3^2} = \sqrt{5^2 \cdot (2^4 + 3^2)} = \sqrt{5^2 \cdot (16+9)}$

$= \sqrt{5^2 \cdot 25} = 5 \cdot 5 = 25.$

c) $\sqrt{3^2 \cdot 5^2 + 2^6} = \sqrt{9 \cdot 25 + 64} = \sqrt{289} = 17$

$\sqrt{2^4 + 2^3 \cdot 3 + 3^2} = \sqrt{16 + 24 + 9} = \sqrt{49} = 7.$

d) $\sqrt{3^4 \cdot 13^2 - 3^4 \cdot 5^2} = \sqrt{3^4 (13^2 - 5^2)} = \sqrt{3^4 \cdot (169 - 25)} = \sqrt{3^4 \cdot 144} = 9 \cdot 12 = 108$

$\sqrt{5^2 \cdot 17^2 - 2^6 \cdot 5^2} = \sqrt{5^2 (17^2 - 2^6)} = \sqrt{5^2 \cdot (289 - 64)} = \sqrt{5^2 \cdot 225} = 5 \cdot 15 = 75.$